Noisy measurements produce faster results

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Abstract

We consider the problem of a finite-time measurement using the Lindblad equation to find analytical results, valid for weak system-environment coupling, obtained for a two-level system in contact with a measurer (Markovian interaction) and a thermal bath (non-Markovian interaction). Analysing the behavior of the coherences we obtained an expression for the maximal duration of the measurement which coincides with the case of no system-environment interaction. The validity of the expression is confirmed by the behavior of the coherences in the case of strong system-environment coupling, found numerically, showing that the environmental noise contributes to a faster measurement.

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I. INTRODUCTION

In quantum mechanics, the state (or wave function) of a system can evolve in two distinct ways: unitarily, according to Schrödinger's equation, when no measurement is being made; or non-unitarily, when a measurement is made on the system, with the reduction of the wave function in one of the eigenstates of the observable. [1, 2] It is exactly in the second case that resides the polemical trait of the quantum theory, as it makes only statistical predictions about the results of the measurement. John von Neumann discussed this problem broadly in his classic book [1], where he admits that the statistical character of the measurement cannot be omitted. To him, the measurement involves necessarily the interaction between the system whose state we wish to determine, and a measuring apparatus whose state is completely known, so that there will be a transference of information between the system and the apparatus. Fundamentally, according to von Neumann, the measurement provides information about the system indirectly through the apparatus, which, after the interaction, is in a superposition of states related to the different eigenstates (and eigenresults) of the main system, then requiring the reduction postulate to determine the probabilities of obtaining a certain value. Proceeding with this reasoning, Asher Peres presents a view [3] where the procedure to obtain information from the system, called *intervention*, is divided in two parts: the *measurement*, when the apparatus interacts with the system and acquires information, and the *reading* (or *output*), when the result of the intervention is made known and the reduction occurs, and when we then obtain the probabilistic information from the diagonal elements of the density matrix - the populations. Thus, the intrinsically statistical character of quantum mechanics is related to the reading.

Supposing that the system-measurer interaction can be analysed with Schrödinger's equation (or, more precisely, with the Liouville-von Neumann equation)[1, 2], using a Hamiltonian that takes into account the main system, the measuring device, and the *Markovian* interaction between the two, it is found an equation where, to the Liouvillian - referring to unitary evolutions - it is added a new term, the Lindbladian - referring to non-unitary processes. This is Lindblad's equation [4, 5], originally obtained in a more general context of quantum dynamical semi-groups [6] for the analysis of irreversible phenomena, but that is used to treat the measurement process too [3, 7]. In particular, in Ref. [3] Peres cites other works about the derivation of the Lindblad equation in the measurement context.

When applied to the measurement process, the Lindblad equation allows something von Neumann did not treat explicitly: to introduce time in the system-measurer interaction. The result can be known by applying the reading to the final density operator, thus maintaining the statistical character of the measurement.

It is our aim here to present results regarding the time evolution of the measurement process, in a very simple illustration, when the system being investigated by measurements is not isolated from environmental perturbations. We use the effective Lindbladian [4, 6] description of the measuring apparatus, whose interaction with the system we assume as Markovian. However, to treat the noise introduced by the fact that, during the finite-duration measurement, the system is perturbed by the environment, we use a non-Markovian Redfield approach. As we do not include the apparatus as part of the environment, we end up formulating an unprecedented hybrid description of a noisy measurement [8, 9]. In our treatment, we start from an equation for the evolution of the total density operator which, before tracing out of the environmental degrees of freedom, involves a unitary evolution of the interaction between the system and its environment, together with a non-unitary Lindbladian evolution of the interaction between the system and the measuring device. Therefore, the degrees of freedom of the device might be thought of as already having been traced out of the formulation, so that the only tracing out left regards the degrees of freedom describing the non-Markovian noise.

In this article we present a simple expression for the maximal duration of the measurement procedure, which is the time the measurement apparatus must be left interacting with the system until the reading can be performed within a minimum error margin. To this end, we will employ the analytical solutions found in Ref. [9], valid for weak system-environment interaction, through the modulus of the coherences, which tend to a value asymptotically close to zero after a certain time. We will consider the time for the system-measurer interaction to end as the instant when the modulus of the coherence reaches a certain small fraction of its original value, because the elimination of the off-diagonal terms is what distinguishes a classical mixture from the pre-measurement quantum-mechanical superposition.

The analytical expression for the maximal value does not depend on the system-environment coupling, only on the system-measurer coupling. As expected, the stronger the system-measurer coupling (*i.e.*, the more intense the measurement), the less time is necessary to complete the reading. Employing a numerical method that allows the analysis of

cases with strong system-environment couplings, we confirmed that the time of measurement decreases as the coupling increases, for the same system-measurer interaction. Therefore, the measurement time is actually reduced by an increase in the noise.

In our studies, we will not consider the process of reduction of the wave function, which displays the statistical character of the intervention. There are interpretations of quantum mechanics [10–12] where the reduction is deemed inexistent. A recent review of this subject can be found in Ref. [13].

This article is structured as follows: in Sec. II we deduce an analytical expression of the measurement time; in Sec. III we analyse the validity of this expression for different system-environment coupling intensities using a numerical method; and we conclude in Sec. IV.

II. MEASUREMENT TIME

A. The hybrid master equation

To derive the hybrid master equation in Ref. [8], we have considered a main system S which, during the measurement process, is interacting with an environment B. Their evolution is governed by the Lindblad equation [4],

$$\frac{d}{dt}\hat{\rho}_{SB}\left(t\right)=-\frac{i}{\hbar}\left[\hat{H},\hat{\rho}_{SB}\left(t\right)\right]+\sum_{j}\left(\hat{L}_{j}\hat{\rho}_{SB}\left(t\right)\hat{L}_{j}^{\dagger}-\frac{1}{2}\left\{\hat{L}_{j}^{\dagger}\hat{L}_{j},\hat{\rho}_{SB}\left(t\right)\right\}\right), \tag{1}$$

where $\hat{\rho}_{SB}\left(t\right)$ is the total density operator, $\hat{H}$ is the total Hamiltonian and the $\hat{L}_j$ are the Lindblad operators that act only on the system. The first term on the right-hand side acting on $\hat{\rho}_{SB}\left(t\right)$ is the Liouvillian superoperator, which accounts for the unitary portion of the propagation, while the second term, the Lindbladian superoperator, represents the Markovian measurement dynamics.

In the Liouvillian term of Eq. (1), the total Hamiltonian can be split in terms $\hat{H}_S$ and $\hat{H}_B$, which act only on S and B, respectively, and an interaction term $\hat{H}_{SB}$:

$$\hat{H}=\hat{H}_B+\hat{H}_{SB}+\hat{H}_S.$$

To model the non-Markovian noise, we suppose that the interaction term that can be decomposed in:

$$\hat{H}_{SB} = \sum_k \hat{S}_k \hat{B}_k, \tag{2}$$

where the $\hat{S}_k$ operate only on the system S, and $\hat{B}_k$, only on the environment B. An interaction of the form given by Eq. (2) is capable of describing both *amplitude-damping* and *phase-damping* quantum channels [14].

The Lindbladian term of Eq. (1) will act solely on the Hilbert space of the system S, since we are interested in measuring system observables only. Using this information about which parts of each superoperator act on which Hilbert spaces, the right-hand side of the Lindblad equation (1) can be split in two commuting superoperators $\hat{\hat{B}}$ and $\hat{\hat{S}}$ that act only on the environment or the system, respectively,

$$\hat{\hat{B}}\hat{X} = -\frac{i}{\hbar}\left[\hat{H}_B, \hat{X}\right], \tag{3}$$

$$\hat{\hat{S}}\hat{X} = -\frac{i}{\hbar}\left[\hat{H}_S, \hat{X}\right] + \sum_j \left(\hat{L}_j \hat{X} \hat{L}_j^\dagger - \frac{1}{2}\left\{\hat{L}_j^\dagger \hat{L}_j, \hat{X}\right\}\right),$$

and an interaction superoperator $\hat{\hat{F}}$, which acts on both Hilbert spaces:

$$\hat{\hat{F}}\hat{X} = -\frac{i}{\hbar}\left[\hat{H}_{SB}, \hat{X}\right]. \tag{4}$$

From this dynamical equation, we have employed the Nakajima-Zwanzig projector superoperator $\hat{\hat{P}}$ [15, 16], defined as

$$\hat{\hat{P}}\hat{X}(t) = \hat{\rho}_B(t_0) \otimes \mathrm{Tr}_B\left\{\hat{X}(t)\right\}, \tag{5}$$

to obtain the hybrid master equation,

$$\frac{d}{dt}\left[\hat{\hat{P}}\hat{\alpha}(t)\right] = \int_0^t dt' \left[\hat{\hat{P}}\hat{\hat{G}}(t)\hat{\hat{G}}(t')\hat{\hat{P}}\hat{\alpha}(t)\right], \tag{6}$$

where

$$\hat{\alpha}(t) \equiv e^{-\hat{\hat{S}}t - \hat{\hat{B}}t}\hat{\rho}_{SB}(t), \tag{7}$$

and

$$\hat{\hat{G}}(t) \equiv e^{-\hat{\hat{S}}t - \hat{\hat{B}}t}\hat{\hat{F}}e^{\hat{\hat{S}}t + \hat{\hat{B}}t}. \tag{8}$$

To obtain the Eq. (6), it is important to emphasize that $\hat{\hat{P}}\hat{\hat{G}}(t)\hat{\hat{G}}(t')\hat{\hat{P}}\hat{\alpha}(0)=0$ - see Ref. [8].

Finally, the reduced density operator $\hat{\rho}_S(t)$, which gives the relevant information about the state of the system, can be found from $\hat{\alpha}(t)$ as defined in Eq. (7):

$$\hat{\rho}_S(t)\equiv \mathrm{Tr}_B\{\hat{\rho}_{SB}(t)\}=e^{\hat{\hat{S}}t}\mathrm{Tr}_B\{\hat{\alpha}(t)\}.$$

B. The specific solutions

In Ref. [9] we have solved the master equation (6) for two different types of measurements. In both cases, we used the following system and environmental Hamiltonians:

$$\hat{H}_S=\hbar\omega_0\hat{\sigma}_z,$$

$$\hat{H}_B=\hbar\sum_k\omega_k\hat{b}_k^\dagger\hat{b}_k, \tag{9}$$

together with a *phase-damping* interaction [14], that is characterized by the following operators in Eq. (2):

$$\begin{cases}\hat{S}_k &= \hbar\hat{\sigma}_z,\\ \hat{B}_k &= g_k\hat{b}_k^\dagger+g_k^*\hat{b}_k,\end{cases}$$

where the $\hat{\sigma}_\alpha$, $\alpha=x,z$ are the Pauli matrices

$$\hat{\sigma}_z=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix},\ \hat{\sigma}_x=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, \tag{10}$$

ω_0 and the ω_k are real constants, $\hat{b}_k$ and $\hat{b}_k^\dagger$ are the annihilation and creation bosonic operators, and the g_k are complex coefficients. The latter are constrained by an Ohmic spectral density,

$$J(\omega)\equiv\sum_k|g_k|^2\,\delta(\omega_k-\omega)=\eta\omega e^{-\frac{\omega}{\omega_c}}, \tag{11}$$

where $\eta\geq 0$ is the constant that gives the strength of the coupling between the system and its environment, and $\omega_c\geqslant 0$ is the cutoff frequency. The initial state of the environment is given by:

$$\hat{\rho}_B = \frac{1}{Z_B}\prod_k e^{-\hbar\beta\omega_k \hat{b}_k^\dagger \hat{b}_k},\ Z_B = \prod_l \frac{1}{1-e^{-\hbar\beta\omega_l}}, \tag{12}$$

where $\beta = (kT)^{-1}$ represents the initial temperature of the bath.

Then, considering the cases of measuring observables $\hat{L} = \lambda\hat{\sigma}_z$ and $\hat{L} = \lambda\hat{\sigma}_x$, we have found the solutions of Eq. (6), valid for weak system-environment interaction η, shown in the next two sections. There, the $\rho_{ij}(t)$, $i, j = 1, 2$, are the matrix elements of the reduced density operator $\hat{\rho}_S(t)$, and the upper indices in brackets in Eq. (15) indicate the basis in which the matrix elements must be taken: the initial conditions are taken from the eigenbasis of $\hat{\sigma}_z$, $\{|+\rangle, |-\rangle\}$, but the final answers are written in the eigenbasis of the measurement, because it is the latter that matters to determine whether the measurement is complete. In the case of measuring $\hat{\sigma}_x$, the basis chosen is composed of its eigenvectors $\{|+\rangle_x, |-\rangle_x\}$, where

$$\begin{cases} |+\rangle_x & = \frac{|+\rangle+|-\rangle}{\sqrt{2}}, \\ |-\rangle_x & = \frac{|+\rangle-|-\rangle}{\sqrt{2}}. \end{cases}$$

1. *The case of $\hat{L} = \lambda\hat{\sigma}_z$ and $T \neq 0$*

In this case, we have found the general solutions

$$\begin{cases} \rho_{11}(t) & = \rho_{11}(0), \\ \rho_{12}(t) & = \rho_{12}(0)\left[\frac{\Gamma\left(\frac{1}{\omega_c\beta\hbar}+i\frac{t}{\beta\hbar}\right)\Gamma\left(\frac{1}{\omega_c\beta\hbar}-i\frac{t}{\beta\hbar}\right)}{\Gamma^2\left(\frac{1}{\omega_c\beta\hbar}\right)}\frac{\Gamma\left(\frac{1}{\omega_c\beta\hbar}+1+i\frac{t}{\beta\hbar}\right)\Gamma\left(\frac{1}{\omega_c\beta\hbar}+1-i\frac{t}{\beta\hbar}\right)}{\Gamma^2\left(\frac{1}{\omega_c\beta\hbar}+1\right)}\right]^{2\eta} e^{-2\lambda^2 t}e^{i2\omega_0 t}. \end{cases} \tag{13}$$

In particular, the expression for the coherence was found after solving the following integral:

$$\rho_{12}(t) = \rho_{12}(0)\exp\left[-4\eta\int_0^t dt'\int_0^\infty d\omega e^{-\frac{\omega}{\omega_c}}\sin(\omega t')\coth\left(\frac{\beta\hbar\omega}{2}\right)\right]e^{-2\lambda^2 t}e^{i2\omega_0 t} \tag{14}$$

2. *The case of $\hat{L} = \lambda\hat{\sigma}_x$, $T = 0$, and $\omega_0 = 0$*

In this case, the particularizations $T = 0$ and $\omega_0 = 0$ were necessary to find the analytical solutions

$$\begin{cases} \hat{\rho}_{11}^{(x)}(t) = & \frac{1}{2} + \mathrm{Re}\left\{\rho_{12}^{(z)}(0)\right\}e^{-8\eta\lambda^2 g_0 t}e^{4\eta\lambda^2[A_-(t)-B_-(t)]}, \\ \hat{\rho}_{12}^{(x)}(t) = & \frac{2\rho_{11}^{(z)}(0)-1}{2}e^{-2\lambda^2 t} - i\mathrm{Im}\left\{\rho_{12}^{(z)}(0)\right\}e^{-2\lambda^2 t}e^{8\eta\lambda^2 g_0 t}e^{-4\eta\lambda^2[A_+(t)+B_+(t)]}, \end{cases} \tag{15}$$

where

$$\begin{cases} g_0 & \equiv \int_0^\infty d\omega \frac{\omega}{4\lambda^4+\omega^2} e^{-\frac{\omega}{\omega_c}}, \\ A_\pm(t) & \equiv 2\int_0^t dt'\; e^{\pm 2\lambda^2 t'} \int_0^\infty d\omega \frac{\omega}{4\lambda^4+\omega^2} e^{-\frac{\omega}{\omega_c}} \cos(\omega t'), \\ B_\pm(t) & \equiv \int_0^t dt'\; e^{\pm 2\lambda^2 t'} \frac{1}{\lambda^2} \int_0^\infty d\omega \frac{\omega^2}{4\lambda^4+\omega^2} e^{-\frac{\omega}{\omega_c}} \sin(\omega t'). \end{cases}$$

As $A_-(t)$ and $B_-(t)$ are only present in the solution for the populations, they will not be necessary for the calculation of the end time of the measurement.

C. Finding the measurement time

As we are dealing with the matrix elements of a density operator, the populations will provide probabilities related to different possible outcomes. We will consider here, in order to establish a criterion for the duration of the measurement, the behavior of the coherences. As it can be seen from the graphs of the moduli of the coherences in both cases, they tend to a value asymptotically close to zero after a short period of time (Fig. 1). Hence, the problem consists in finding a *simple* expression for the time when $\left|\rho_{12}^{(\alpha)}(t)\right|$, $\alpha = x, z$, equals a fraction f of its original value, *i.e.*

$$\left|\rho_{12}^{(\alpha)}(t_M)\right| = f\left|\rho_{12}^{(\alpha)}(0)\right|, \tag{16}$$

where $0 < f < 1$. The non-trivial forms of Eqs. (14) and (15) prevent the exact solution of Eq. (16). Thus, we have to approximate the expression by means of series expansions, considering $t_M \ll 1$, which is justified from the behavior of the two expressions. As both cases involve exponentials,

$$|\rho_{12}(t)| \propto e^{F(t)}, \tag{17}$$

where the $\propto$ sign includes exponentials with linear arguments and $F(t)$ is a function whose form depends on the situation considered. Our approach consists in expanding $F(t)$ in a power series of t :

$$F(t) \;=\; \sum_{m=0}^{\infty} t^m F^{(m)}(0),$$

replacing the expansion in the argument of the exponential

$$\left|\rho_{12}\left(t\right)\right| \propto e^{\sum_m t^m F^{(m)}(0)}, \tag{18}$$

neglecting higher-order terms, and solving Eq. (16) using Eq. (18).

1. *The case of $\hat{L} = \lambda\hat{\sigma}_z$ and $T \neq 0$*

First, we consider a measurement that commutes with the noise, as given in Eq. (14). There, the square modulus of the coherence is:

$$\left|\rho_{12}\left(t\right)\right| = \left|\rho_{12}\left(0\right)\right| \exp\left[-4\eta\int_0^t dt'\int_0^\infty d\omega e^{-\frac{\omega}{\omega_c}}\sin\left(\omega t'\right)\coth\left(\frac{\beta\hbar\omega}{2}\right)\right]e^{-2\lambda^2 t},$$

so that Eq. (16) becomes

$$\left|\rho_{12}\left(0\right)\right| \exp\left[-4\eta\int_0^{t_M} dt'\int_0^\infty d\omega e^{-\frac{\omega}{\omega_c}}\sin\left(\omega t'\right)\coth\left(\frac{\beta\hbar\omega}{2}\right)\right]e^{-2\lambda^2 t_M} \;=\; f\left|\rho_{12}\left(0\right)\right|,$$

or, simplifying,

$$\exp\left[-4\eta\int_0^{t_M} dt'\int_0^\infty d\omega e^{-\frac{\omega}{\omega_c}}\sin\left(\omega t'\right)\coth\left(\frac{\beta\hbar\omega}{2}\right)\right]e^{-2\lambda^2 t_M} \;=\; f.$$

We apply the methodology of the expansion of the argument of the first integral defined by the function

S

$$F\left(t\right) \equiv -4\eta\int_0^\infty d\omega e^{-\frac{\omega}{\omega_c}}\frac{1-\cos\left(\omega t\right)}{\omega}\coth\left(\frac{\beta\hbar\omega}{2}\right). \tag{19}$$

o, up to the first order in t,

$$e^{-2\lambda^2 t_M} = f, \tag{20}$$

and the expression for t_M becomes

$$t_M = -\frac{1}{2\lambda^2}\ln\left(f\right), \tag{21}$$

where $f < 1$.

The expansion up to the first order, therefore, only includes the contribution of the apparatus, being free of the action of the environment. The right-hand side of Eq. (19), however, is non-negative, as the integral consists of non-negative functions. Therefore,

$F(t) \leq 0$, and the noise terms can only make the coherence smaller, decreasing the duration of the measurement by our criterion. It follows that the t_M calculated above is the upper limit for the duration of a noisy measurement. Moreover, it can be noted that the coupling with the environment, η, decreases the value of the coherence, so that a stronger noise always results in a faster end to the measurement.

2. *The case of $\hat{L} = \lambda\hat{\sigma}_x$, $T = 0$, and $\omega_0 = 0$*

Now we consider the second of Eqs. (15). The square modulus of the coherence becomes:

$$\left|\rho_{12}^{(x)}(t)\right|^2 = \left[\frac{2\rho_{11}^{(z)}(0) - 1}{2}\right]^2 e^{-4\lambda^2 t} + \mathrm{Im}\left\{\rho_{12}^{(z)}(0)\right\}^2 e^{-4\lambda^2 t} e^{16\eta\lambda^2 g_0 t} e^{-8\eta\lambda^2[A_+(t)+B_+(t)]}. \quad (22)$$

Developing Eq. (16), we find:

$$\frac{R_0}{N_0} e^{-4\lambda^2 t_M} + \frac{I_0}{N_0} e^{-4\lambda^2 t_M} e^{16\eta\lambda^2 g_0 t_M} e^{-8\eta\lambda^2[A_+(t_M)+B_+(t_M)]} = f^2,$$

where

$$\begin{aligned} R_0 &= \left[\frac{2\rho_{11}^{(z)}(0) - 1}{2}\right]^2, \\ I_0 &= \mathrm{Im}\left\{\rho_{12}^{(z)}(0)\right\}^2, \\ N_0 &= R_0 + I_0. \end{aligned}$$

For the sake of simplicity, we define:

$$\begin{aligned} k_1 &= \frac{R_0}{N_0}, \\ k_2 &= \frac{I_0}{N_0}, \end{aligned}$$

so that the main equation of our problem becomes:

$$k_1 e^{-4\lambda^2 t_M} + k_2 e^{-4\lambda^2 t_M} e^{16\eta\lambda^2 g_0 t_M} e^{-8\eta\lambda^2[A_+(t_M)+B_+(t_M)]} = f^2. \quad (23)$$

We apply the expansion over the last exponential of the second term on the left-hand side,

$$F(t) = -8\eta\lambda^2 \left[A_+(t) + B_+(t)\right], \tag{24}$$

then, we have in Eq. (23):

$$k_1 e^{-4\lambda^2 t_M} + k_2 e^{-4\lambda^2 t_M} e^{16\eta\lambda^2 g_0 t_M} e^{-8\eta\lambda^2 (2g_0 t_M)} = f^2,$$

hence

$$\frac{k_1 + k_2}{f^2} = e^{4\lambda^2 t_M}.$$

Therefore, according to the definitions of k_1 and k_2,

$$t_M = -\frac{1}{2\lambda^2} \ln(f), \tag{25}$$

keeping in mind that $f < 1$. This expression is identical to Eq. (21), found in the previous item.

Moreover, it is possible to rewrite the second of Eqs. (15) in the form:

$$\hat{\rho}_{12}^{(x)}(t) = \frac{2\rho_{11}^{(z)}(0) - 1}{2} e^{-2\lambda^2 t} - i\mathrm{Im}\left\{\rho_{12}^{(z)}(0)\right\} \exp\left\{-2\lambda^2 t - 8\eta \int_0^t dt' \int_0^\infty d\omega\omega\lambda^2 \frac{e^{2\lambda^2 t'} \cos(\omega t') - 1}{4\lambda^4 + \omega^2} e^{-\frac{\omega}{\omega_c}}\right\}$$
$$\times \exp\left\{-4\eta \int_0^t dt'\, e^{2\lambda^2 t'} \frac{1}{\lambda^2} \int_0^\infty d\omega \frac{\omega^2}{4\lambda^4 + \omega^2} e^{-\frac{\omega}{\omega_c}} \sin(\omega t')\right\}.$$

As we are dealing with small perturbations caused by the environment, it is safe to assume that the exact measurement time will be much shorter than the typical decoherence time, so that, in the time periods we are dealing with, $t_M \ll \omega_c^{-1}$. Therefore, we can consider that the $\omega t'$ in the integrals is close to zero, thus leading to the approximations $\cos(\omega t') \approx 1$ and $\sin(\omega t') \approx \omega t'$, thus guaranteeing the non-negativity of the two integrands during the characteristic time of the measurement. We will have, therefore, negative numbers multiplying the coupling constant η, thus showing that an increase in the coupling with the environment increases the speed with which the measurement ends even if the noise is causing decoherence in a different basis.

D. Final expression

We have found that, regardless of the observable measured, $\hat{L} = \lambda\hat{\sigma}_x$ or $\hat{L} = \lambda\hat{\sigma}_z$ (even though some particularizations - $\omega_0 = 0$, $T = 0$ - were made in the first case), we have the same expression for the upper limit of the time of measurement:

$$t_M = -\frac{1}{2\lambda^2} \ln\left(f\right). \tag{26}$$

This expression does not depend on the system-environment coupling, η, nor on initial conditions, but only on the system-measurer coupling λ. The initial conditions are not found to affect the time of measurement, but the demonstrations above show that an increase in η makes the measurement faster in the two bases.

III. COMPARISONS

In this section, we compare the upper limit obtained above with cases where the duration of measurement is shorter. We consider that the phase noise occurs while the observable $\hat{\sigma}_x$ is being measured. The following numerical results were obtained according to the superoperator-splitting method described in Ref. [9], to which it is applied the condition from Eq. (16). In all the simulations, we have chosen the initial state $\frac{1}{\sqrt{2}}\left(|+\rangle - e^{i\pi/4}|-\rangle\right)$, so that the coherences have initially no real part:

$$\rho_{12}^{(x)}(0) = \frac{1}{2}\left(1 - e^{i\pi/4}\right)\frac{1}{2}\left(1 + e^{-i\pi/4}\right) = -\frac{1}{2}\sin\left(\frac{\pi}{4}\right)i = -\frac{1}{2\sqrt{2}}i.$$

Simulations of this part of the coherence are shown in Fig 1.

This initial condition requires the simulation of only $\mathrm{Im}\left\{\rho_{12}^{(x)}(0)\right\}$, which must satisfy

$$\frac{\mathrm{Im}\left\{\rho_{12}^{(x)}(t_M)\right\}}{\mathrm{Im}\left\{\rho_{12}^{(x)}(0)\right\}} \leq f$$

at the end of the measurement, at instant t_M. This simplified condition to assess t_M is employed in Fig. 2, where it can be verified that an increase in η or λ makes the measurement process faster. In these graphs, it can be seen that the increase in the measurement time also occurs for large values of η, as expected from the analytical results.

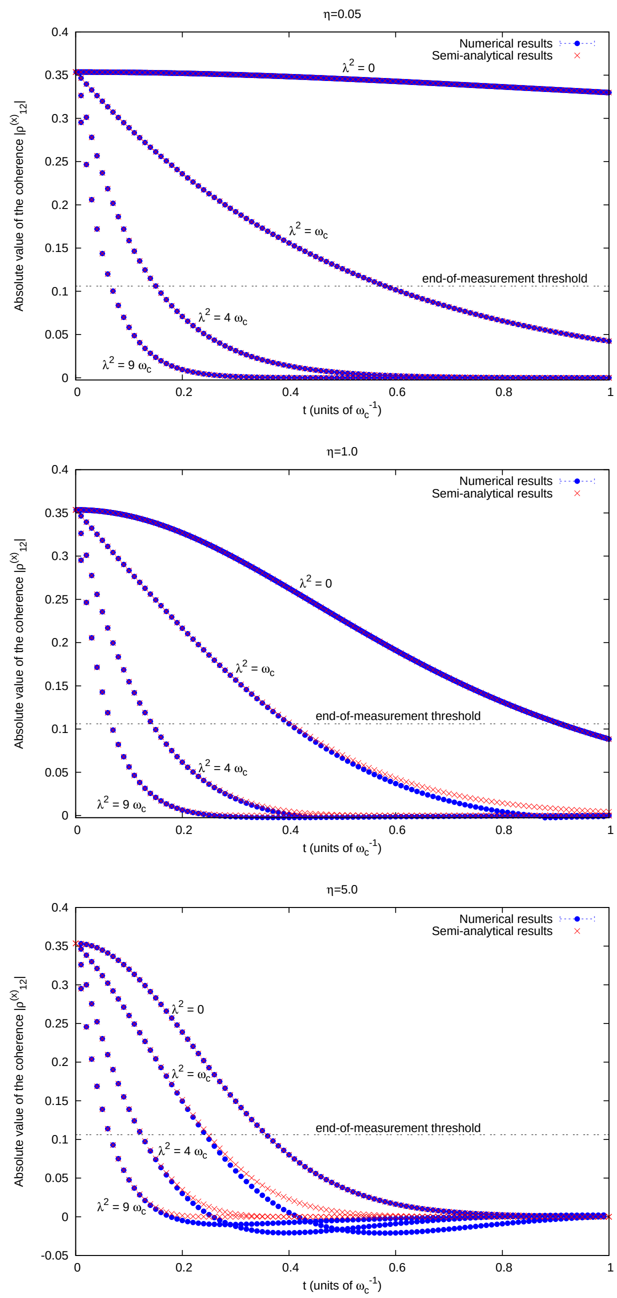

Figure 1: (Color online and in black-and-white in print) Time evolution of the absolute value of the coherence for the initial state $\frac{1}{\sqrt{2}}\left(|+\rangle - e^{i\pi/4}\,|-\rangle\right)$, for different strengths of measurement (λ) and both weak ($\eta = 0.05$) and strong ($\eta = 1, \eta = 5$) couplings with the environment. The numerical results are found according to Ref. [9], while the semi-analytical results are those found in Eq. (15), so that there is a better agreement between the two methods when the noise is not so intense. Choosing $f = 0.3$ to define the end of the measurement, it can be seen from these curves that the measurement is faster when the coupling with the apparatus is stronger (λ increases) or when the noise is more intense (greater η), confirming the analytical predictions.

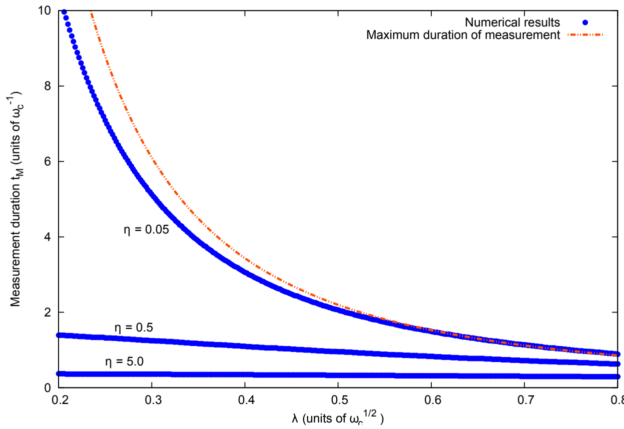

Figure 2: (Color online and in black-and-white in print) Numerical results for the duration of the measurement, using $f = 0.3$, as a function of the coupling with the apparatus (λ), for different values of η. The dashed line represents the upper limit to the duration of measurement, given by Eq. (26). From these curves, it can be seen once again that both an increase in the noise or in the strength of measurement can decrease the duration of the measurement process.

IV. CONCLUSIONS

The two expressions for the maximal duration of the measurement obtained analytically, Eqs. (21) and (25), are identical. This is an interesting fact, given that the situations were distinct, not only because of the different types of measurements being made, but also because, in the second case, particularizations were made.

Even less intuitive is the fact that the measurement is performed faster when a source of noise is present. This fact would not be so surprising when the interaction Hamiltonian and the measurement observable do commute, as in this case both effects cause decoherence on the same basis. More surprising is the fact that the effect still occurs when the measurement and the noise are incompatible, hinting on the universality of such phenomena. At least in this simple two-state system where we have shown it to occur, this phenomenon could possibly be used to create more precise measurements by finding a balance between the detrimental effects that the environment has on the state of the system and the beneficial effects it has in improving the measurement time.

There are studies on the problem of the *reading* (or *output*) *time* [17, 18], where this time was considered as a constant of nature, independent of the system under scrutiny. In this way, *if* the process of reduction of the wave function does exist (contrary to the

Everettian thesis [10–12]), a complete treatment for the *intervention* problem should include our measurement time - Eq. (26) - plus the reading time. Anyway, the approach of this paper does not contradict the statistical character of quantum mechanics.

Of course, it all depends on the validity of the measurement time as proposed in this Lindbladian treatment of the measurement apparatus. As our expression for the upper limit in the measurement time is not only simple, but also depends solely on the system-measurer coupling, it can be empirically tested. Furthermore, our method can be employed in more complex and, perhaps, more realistic systems (more than two levels, other types of system-environment interaction, other types of environments, etc.), with results that can be used in comparative studies against other quantum-measurement approaches, such as the thermodynamic one of references [19–21]. Then, it may ellucidate which model best describes the real world and contribute to a better understanding of the fundamentals of quantum mechanics.

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